The Effectiveness of Occasionally-Used Paper Ballots

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Several critics of voter-verified paper ballots have claimed that a paper ballot is useless if only a small portion of voters bother to verify its correctness. If the accuracy of a large tally – the end-of-day totals, in our case – cannot be verified by a spot check of a small sample – a portion of the paper ballots – then the entire public polling industry is in for quite a shock. A sample of a few thousand people is often used to gauge the opinion of several million, so why not use a portion of the paper ballots to verify the correctness of the count?

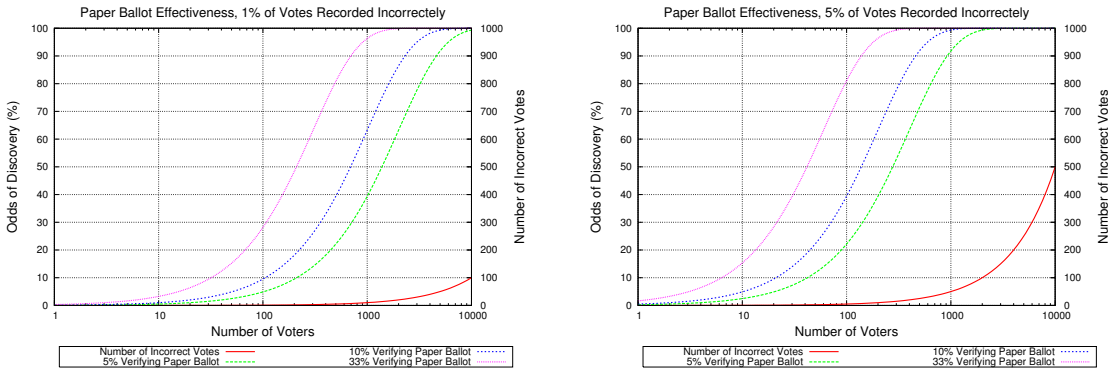
So just how effective **is** a voter-verified paper ballot? Well, in addition to serving as the ballot of record, even when the electronic record is lost, we can calculate the odds that a malfunctioning machine will be discovered by the voters who are verifying their ballot correctly. We say "verifying their ballot correctly" since it has come to our attention that it is possible for the voter to miss an error on a printed ballot.

Here is the set-up for our calculation. In it, we have

- N, the number of voters who cast ballots on the malfunctioning machine;
- M, the percentage of votes that the machine is modifying, either due to an honest error in its program or screen calibration, or through malicious tampering (an attacker);
- V, the percentage of voters who manage to verify their paper ballot correctly;
- D, the odds that at least one of the voters will discover a discrepancy between what they entered into the computer and the paper ballot they are given;
- C, the number of ballots that are actually changed.

Given the definitions of N, M, and V, we can calculate D and C as

$$\begin{aligned} D &= 100 \cdot (1 - (1 - (M \cdot V))^N) \\ C &= N \cdot M \end{aligned}$$



(a) 1% mis-recorded or modified.

(b) 5% mis-recorded or modified.

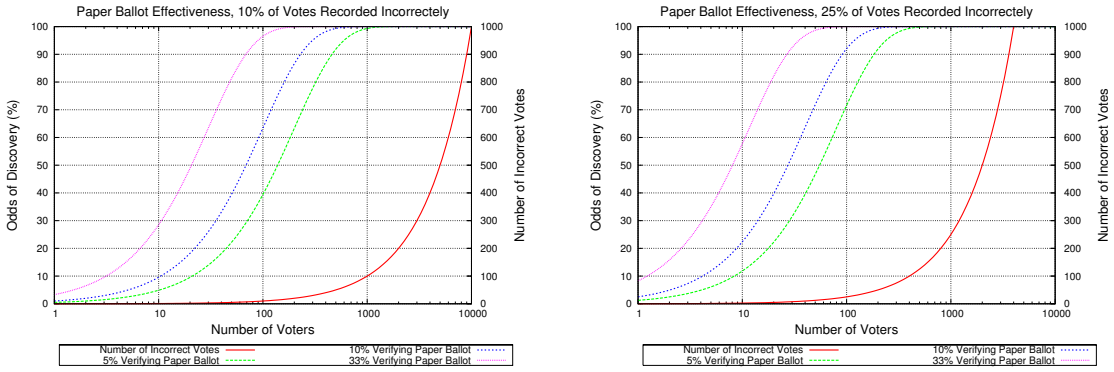
Figure 1: Machine mis-records or modifies a small number of the votes

Here, we graph N on the X-axis, D on the left-hand Y-axis, and C on the right-hand Y-axis. In figure 1(a), M is set to 1%: the machine makes a mistake on a random 1% of the ballots. The way we read these graphs is by looking at a given number of voters, and reading the odds that at least one of them will have noticed a problem, and how many votes will have been changed by that point.

For example, after 1,000 voters the machine has probably changed 10 votes (our 1%). Even if only 5% of the voters – 1 out of 20 – verify their ballot correctly, there is 40% chance that someone has noticed something is wrong. If 1 out of 10 verify theirs correctly, there is about a 2 in 3 chance that someone has caught the machine making a mistake. If 1 out of 3 voters manage to verify their ballot correctly, there is over a 96% chance that someone has noticed something is wrong.

For a large county like Guilford County, NC – with over 200,000 voters taking part in the last election – even if only 1 out of 100 people examine their ballot for mistakes, there is only a **1 in 587 million** chance that the problem will escape notice. And with the machine making a mistake on 1 out of every 100 ballots, that means only 2,000 votes will be incorrect; less than half of the number of votes lost in Carteret County in 2004.

Figure 1(b) shows a situation in which the machine is making a mistake on 5% of the votes. By the time 1,000 people have cast their votes, there is over a 90% chance that the mistake will be caught. If 1 out of 3 people verify their ballot correctly, there is only a **1 in 16 million** chance the the problem will go unnoticed.



(a) 10% mis-recorded or modified.

(b) 25% mis-recorded or modified.

Figure 2: Machine mis-records or modifies a larger number of the votes

An investigation in Pennsylvania discovered that the UniLect Patriot DRE is unreliable and unable to count votes accurately. This is the same model of machine that lost over 4,500 votes in Carteret County, NC. It is also used in Burke County, NC, which recorded an unusually high percentage of voters who went to the polls but did not cast votes, even in state-wide and federal races (Governor, U.S. Senate, President, etc). It is possible that up to 10% of the votes in Burke County were recorded incorrectly or not at all.

As shown in figure 2(a), if Burke had used voter-verified paper ballots, even if only 1 out of 20 people verified them correctly, there is a 99.6% chance that the mistakes would have been caught after only 1,000 votes.

In short, even if a paper ballot is used infrequently and verified correctly a small percentage of the time, is leads to an incredibly high likelihood that machine errors will be caught. The cost-benefit tradeoffs of a voter-verified paper ballot trail are tremendous.